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SCHRÖDINGER QUANTUM EQUATION FROM CLASSICAL AND QUANTUM HARMONIC OSCILLATOR

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ABSTRACT

A new quantum model that accounts for the medium friction is derived. The first advantage is the addition of natural oscillation of particle. The second advantage is the incorporation of frication effect in the Hamiltonian operator. This means that both Schrödinger equation and energy Eigen equation are affected by friction. The Eigen energy is not affected by friction, which is in direct conflict with experiment and common sense.

INTRODUCTION

Quantum Frictional Oscillating and Boltzmann Disruption Law

The description of particles and waves moving inside a resistive medium for quantum systems plays important role in superconductivity and super fluids, as well as nano system [1,2,3]. The theoretical models that are developed to describe quantum friction; the quantum treatment of this problem is based on scattering collision theories which are very complex. Thus one needs new quantum approach that van simply tackles friction effects of the medium, Different attempts wave made to construct such model the most promising are in that based on relaxation. In this model the solutions of quantum equation lead to radioactive decay law and transport probability. Thus one need a well-defined framework performs this task. This can be done, in this chapter by utilizing Maxwell's equation and plasma equations.

The time attenuation coefficient from Maxwell's equation

Maxwell's theory is one many theories that can describe electromagnetic field. The equation of the electric field intensity E inside a polarized medium field, Takes the form [see equation (2-3-1-18)]:

$$-\nabla^2 E + \mu_o \sigma \frac{\partial E}{\partial t} + \mu_o \varepsilon_o \frac{\partial^2 E}{\partial t^2} = -\mu_o \sigma \frac{\partial^2 P}{\partial t^2} \qquad (2.1)$$

The solution of this equation can be expressed in terms of time attenuation coefficient α, wave number K, and angular frequency ω, to be in the form [4]

$$E = E_0 e^{-\alpha t} e^{(kx-\omega t)} \qquad (2.2)$$

The displacements of charges can be described by:

$$E = E_0 e^{-\alpha t} e^{(kx-\omega t)} = \frac{X_0}{E_0} E \quad (2.3)$$

The electric dipole moment can thus define in terms of charge density:

$$\rho = -e\, n_e X = -e\, n_e X_o e^{-\alpha t} e^{/(Kx-\omega t)\alpha t} = \frac{n_{ec} X_o}{E_o} E \qquad (2.4)$$

(1)Differentiating (2.2) with respect to X and t yields:

$$\frac{\partial E}{\partial x} = iKE \ , \ \nabla^2 E = \frac{\partial^2 E}{\partial x^2} = -K^2 E$$

$$\frac{\partial E}{\partial t} = (-\alpha - i\omega)E \ , \frac{\partial^2 E}{\partial t^2} = (-\alpha - i\omega)^2 \qquad (2.5)$$

(2) Differentiating (2.2) with respect to t gives:

$$\frac{\partial P}{\partial x} = -e\, n_{elec} \qquad = \frac{\partial x}{\partial t} \quad = i - e\, n\omega x$$

$$\frac{\partial^2 E}{\partial x^2} = e\, n_{elec}\,\omega^2 \ x = \ e\, n_{elec}\,\omega^2 x \qquad (2.6)$$

Inserting equations (2.5), (2.6) in equation (2.1) yield:

$$\left[K^2 E + \mu\varepsilon(-\alpha - \omega)^2 E + \mu\sigma(-\alpha - i\omega)E\right] = \frac{-e\, n\ \omega^2 \mu x_o}{E_o} E$$

$$K^2 + \mu\varepsilon\alpha^2 - \omega\left(\alpha^2 - \omega^2 + 2i\alpha\omega\right) - E + \mu\sigma\alpha - \mu\sigma\omega i = \frac{-e\, n\ \omega^2 \mu x_o}{E_o} \quad (2.7)$$

Thus equation (2.7) reads:

$$K^2 + \frac{\left(\alpha^2 - \omega^2 + 2i\alpha\omega\right)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i \ \ \frac{-e\, n\ \omega^2 \mu x_o}{E_o} E = -\frac{\mu\partial^2 P}{t^2}$$

$$K^2 + \frac{\left(\alpha^2\right)}{c^2} - \frac{\omega^2}{c^2} + \frac{(2i\alpha\omega)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i = \frac{-e\, n\ \omega^2 \mu x_o}{E_o}$$

But since the speed of light is given by:

$$\mu\varepsilon = \frac{1}{c^2}$$

Thus equation (2.7) reads

$$K^2 + \frac{\left(\alpha^2 - \omega^2 + 2i\alpha\omega\right)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i \ \ \frac{-e\, n\ \omega^2 \mu x_o}{E_o} E = -\frac{\mu\partial^2 P}{t^2} \quad (2.8)$$

$$K^2 + \frac{\alpha^2}{c^2} - \frac{\omega^2}{c^2} + \frac{(2i\alpha\omega)}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i = \frac{-e\, n\ \omega^2 \mu x_o}{E_o}$$

But:

$$K^2 = +\left(\frac{2\pi}{\lambda}\right) = \left(\frac{2\pi f}{\lambda f}\right)^2 = \frac{\left(\omega^2\right)}{c^2}$$

Thus:

$$\frac{\alpha^2}{c^2}+\frac{2i\alpha\omega}{c^2} - \mu\sigma\alpha - \mu\sigma\omega i \quad \frac{-e\,n_e\ \omega^2\mu x_o}{E_o}\ \omega^2 \qquad (2.9)$$

Equation real parts on both sides on (2.9)

$$\frac{\alpha^2}{c^2} - \mu\sigma\alpha = \frac{-e\,n_e\ \mu x_o}{E_o}\ \omega^2 \qquad (2.10)$$

Since the speed of light is given large $c >> \alpha$

Hence:

$$\frac{\alpha^2}{c^2} \to 0 \qquad (2.11)$$

Thus the absorption coefficient is given by:

$$\alpha = \frac{-e\,n_{elec}\ x_o}{E_o} \quad \frac{\omega^2}{\sigma} \qquad (2.12)$$

Expression (12) can be simplified by using the conductivity expression for direct current:

$$\sigma = \frac{ne^2\tau}{m} \qquad (2.13)$$

And by using the electron equation under the action of electromagnetic field, where:

$$m\ddot{x} = -\ \omega_n^2 x_o\ e^{i\omega e} = -e\ x_o\,e^{i\omega t}$$

Thus:

$$\frac{x_o}{E_o} = \frac{e}{m\,\omega^2} \qquad (1.14)$$

Thus a direct substitution of (2.13) and (2.14) in (2.12) yields:

$$\alpha = \frac{e\,n\,\omega^2 m}{n\,e^2\tau}\left(\frac{e}{m\,\omega^2}\right) = \frac{1}{\tau} \qquad (2.15)$$

One can also engorge polarization term in the equation (a) to get:

$$\frac{\alpha^2}{c^2} - \frac{2i\alpha\omega}{c^2} - \mu\sigma\omega i - \mu\sigma\alpha = 0 \qquad (2.16)$$

Since for osculating electron the equation of motion is:

$$m\ddot{x} = -\ \omega_n^2 x_o\ e^{i\omega t} = e\,E_o\,e^{i\omega t} - \frac{m v_o e^{i\omega t}}{\tau} \qquad (2.17)$$

Since $\quad v = dx/dt = i\omega x \qquad (2.18)$

Hence:

$$\left[i\ \omega + \frac{1}{\tau}\right] m v = e\ E \qquad (2.19)$$

Using the relation:

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$$J = nev = \frac{ne^2}{m}\left[i\,\omega + \frac{1}{\tau}\right]^{-1} E = \sigma E\left[\sigma_1 + i\,\sigma_2\right]E^2 \qquad (2.20)$$

Therefore:

$$\sigma_1 = \frac{ne^2(T)^{-1}}{m}\left[(T)^{-1} + \omega^2\right] \quad , \quad \sigma_2 E = \frac{ne^2\omega}{m}\left[\tau^{-1} + \omega^2\right]^{-1} \qquad (2.21)$$

Inserting the complex conductivity of (20) in equation (16) yields:

$$\frac{\alpha^2}{c^2} - \frac{2i\alpha\omega}{c^2} - \mu\left(\sigma_1 + i\sigma_2\right)\alpha - \mu\left(\sigma_1 + i\sigma_2\right)\omega i = 0 \qquad (2.22)$$

Equating imaginary parts one gets:

$$\alpha = \left[\frac{2\,\omega}{c^2} + \mu\,\sigma_2\right] = -\mu\,\sigma_1\,\omega \qquad (2.23)$$

Using C^2 is very large, one can neglect the first term in the bracket, 1.e:

$$2\,\omega / c^2 \rightarrow \qquad (2.24)$$

To get:

$$\alpha = \frac{-\sigma_1\omega}{\sigma_2} \qquad (2.25)$$

In view of in equation (2.24).The absorption coefficient (2.25) is given by:

$$\alpha = \frac{1}{\tau} \qquad (2.26)$$

ABSORPTION COEFFICIENT AND RELAXATION TIME FOR POLARIZED AND NON POLARIZED MATERIAL:

Consider an electron osculating naturally with frequency w_o and affected by the osculating electric field[5,6]:

$$E = E_o\, e^{i\,\omega t} \qquad (3.1)$$

The equation of motion for such electron is given by:

$$m\frac{dv}{dt} = -\,kx + eE - \frac{mv}{\tau} \qquad (3.2)$$

Where:

$$K = m\,\omega_o^2 \qquad X = x_o\, e^{i\,\omega t} \qquad V = i\,\omega x \quad (3.3)$$

There for equation (   ) becomes:

$$i\;m\,\omega v = -m\,\omega_o^2\, x + eE\;\frac{-m\,v}{\tau} = i\;m\,\omega_o\, v\,\frac{-m\,v}{\tau} + eE$$

$$\left[i\left(\omega - \omega_o\right) + \frac{1}{\tau}\right] m\,v = eE \qquad (3.4)$$

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By setting:

$$\omega-\omega_o = \Delta\,\omega \qquad (3.5)$$

Thus:

$$V = \frac{e\,E}{m\left[i\,\Delta\omega+\frac{1}{\tau}\right]} = \frac{e\left[\frac{1}{\tau}-i\,\Delta\,\omega\right]E}{m\left[i\,\Delta\omega+\frac{1}{\tau}\right]} \qquad (3.6)$$

According to the relation between current density, velocity and conductivity one gets:

$$J = nev = \frac{ne^2\left[\frac{1}{\tau}-i\,\Delta\omega\right]}{m\left[\Delta\omega^2+\frac{1}{\tau^2}\right]} \qquad (3.7)$$

Hence the conductivity is given by:

$$\sigma = \frac{ne^2\left[\frac{1}{\tau}-i\,\Delta\omega\right]}{m\left[\Delta\omega^2+\frac{1}{\tau^2}\right]} \qquad (3.8)$$

For non-polarized medium, equation (2.1) can be solved by assuming:

$$E = E_o\, e^{i(kx-\omega t)}$$

To get:

$$K^2-\frac{\omega^2}{c^2}+i\mu_0\omega\sigma = 0$$

In view of this equation beside (3.8) one gets:

$$K^2-\frac{\omega^2}{C^2} = \mu\sigma\omega i = \frac{\mu_0\omega\left[\Delta\omega+\frac{i}{\tau}\right]}{m\left[\Delta\omega^2+\frac{1}{\tau^2}\right]}ne^2 \quad (3.9)$$

If the wave length in the medium is near to that of free space it follows that:

$$K^2 = \left(\frac{2\pi}{\lambda}\right)^2 = \left(\frac{2\pi f}{\lambda f}\right)^2 = \frac{\omega^2}{C^2} \qquad (3.10)$$

As a result:

$$K^2-\frac{\omega^2}{C^2} = 0 \qquad (3.11)$$

Hence using equation (3.11) in equation (3.9) yields:

$$\omega-\omega_0=-\Delta\omega=\frac{-i}{\tau}\ \omega_0-\omega=\frac{i}{\tau} \quad (3.12)$$

A similar result can be found for polarized material According to equation (2.3):

$$X=x_o e^{i\omega t} \qquad \dot{X}=v=-i\omega x_o e^{-i\omega t}=V_o e^{-i\omega t}$$
$$\ddot{X}=--i\omega v \qquad P=e\,n\,x \quad (3.13)$$

Thus:

$$\mu\frac{\partial^2 P E}{\partial t^2}=\mu e n\ddot{x}=-i\ \omega\ \mu\ nev\ =-i\ \omega\ \mu\ \ J=-i\ \omega\ \mu\sigma\ E$$

Thus equation (2.8) for α neglected becomes:

$$K^2-\frac{\omega^2}{C^2}=\mu\sigma\omega i=0$$

This is typical equation (3.9) thus gives again:

$$\omega_o-\omega\ =\frac{i}{\tau} \quad (3.14)$$

According to plank hypothesis the original energy and the energy for frictional medium are given by:

$$E_f=E_o-E=\hbar\ \omega=\left(\frac{\hbar}{\tau}\right)i \quad (3.15)$$

This expression for frictional energy agrees with eqn (2.26) and (2.2)

QUANTUM EQUATION FOR FRICTIONAL MEDIUM

In view of equation (2.2) in a resistive medium with the aid of eqn (2.2) one can write the wave function ψ similar to E. This is justifiable as far as[7,8]

$E^2\alpha$ Number of photon

$\psi^2\alpha$ Number of particles

Thus, one can write ψ to be:

$$\psi=A\ e^{\frac{i}{\hbar}\left(E-\frac{i\hbar}{\tau}\right)t} \quad (4.1)$$

$$\frac{\partial\psi}{\partial t}=\frac{-i}{\hbar}\left(E-\frac{i\hbar}{\tau}\right)\psi$$

$$i\hbar\frac{\partial\psi}{\partial t}+\frac{i\hbar}{\tau}\psi=E\psi$$

$$i\hbar\left[\frac{\partial}{\partial t}+\frac{1}{\tau}\right]\psi=E\psi \quad (4.2)$$

$$\hat{H}\ \psi=E\psi \quad (4.3)$$

Thus the energy operator takes the form:

$$\hat{H}\ \psi=E\psi \quad (4.4)$$

Using:

$$E = \frac{P^2}{2m} + V \qquad (4.5)$$

$$E\psi = \frac{P^2}{2m}\psi + V\psi \qquad (4.6)$$

$$\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} p \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{-p}{\hbar^2}\psi$$

In 3 dimensions:

$$-\hbar^2 \nabla^2 \psi = p^2 \psi \qquad (4.7)$$

$$i\hbar \left[\frac{\partial}{\partial t} + \frac{1}{\tau}\right]\psi = \frac{-\hbar^2}{2m}\nabla^2 \psi + V\psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{-\hbar^2}{2m}\nabla^2 \psi + V\psi - \frac{i\hbar}{\tau}\psi \qquad (4.8)$$

STRING OSCILLATORY SOLUTION

To find solution for harmonic oscillator, it is important to separate variables. Thus one can write the wave function[9]

$$\psi(r,t) = f(t)\,u(r) \qquad (5.1)$$

Inserting of equation (5.1) in (4.8) yields:

$$\frac{i\hbar \partial f}{f \partial t} = \frac{\hbar}{2mu}\nabla^2 u + V = \frac{i\hbar}{\tau} + E_0 \quad (5.2)$$

Therefore:

$$i\hbar \frac{\partial f}{\partial t} = E_o f \qquad (5.3)$$

The solution of this equation is:

$$f = A_o e^{-i\beta_o t} \qquad (5.3)$$

Substituting (5.4) in (5.3) yields:

$$\hbar \beta_o = E_o \qquad (5.5)$$

For Harmonic Oscillator the potential is given by:

$$V = \frac{1}{2} k x^2 \qquad (5.6)$$

This equation (5.2) reduces to:

$$\frac{-\hbar^2}{2m}\nabla^2 u + \frac{1}{2} k x^2 u = \left(E_o + \frac{i\hbar}{\tau}\right) u = E u \qquad (5.7)$$

But the energy of harmonic Oscillator is given by:

$$E = E_o + \frac{i\hbar}{\tau} = \left(n + \frac{1}{2}\right)\hbar \omega \qquad (5.8)$$

$$n = 1, 2, 3, \ldots\ldots$$

The harmonic Oscillator satisfies periodicity condition, 1 .e:

$$f\ (t + T\ )\ = f\ (t\ ) \qquad (5.9)$$

In view of equation (5.4) this requires:

$$e^{-i\ \beta_o T} = \cos\ \beta_o T\ - i\ s\text{in}\ \beta_o T\ = 1$$

This means that:

$$\cos\ \beta_o T\ = 1 \qquad \sin\ \ \beta_o T\ = 0$$

Hence:

$$\beta_o T\ \ = 2\ \pi\ \ s$$

$$S\ = 1, 2, 3, \ldots\ldots$$

$$\beta_o\ = \frac{2\ \pi}{T} s \qquad s\ \omega \qquad (5.10)$$

Inserting (5.10) in (5.5), the energy is given by:

$$E_o = \hbar\ \omega s \qquad (5.11)$$

This energy lost by friction is thus gives according to eqn (5.8) and (5.11) given by:

$$E_f\ = E - E_o = \hbar\ \omega \left(n - s + \frac{1}{2} \right) \qquad (5.12)$$

DISCUSSION

Maxwell equation for electric field is used to find a useful expression for absorption coefficient was found. The solution suggested includes damping term α . The relation between polarization and displacement , together with the expression of conductivity for direct current , in addition to the eqn of electron motion in the presence of electric field and frictions , are all used to find α . It is found also that α is equal to the reciprocal of relaxation time τ .

Using the electron equation of motion, together with conductivity relation of alternating current, beside Maxwell equation solution for travelling an attenuated wave in section(3) a useful expression for absorption coefficient is found also. According to eqn(3.6) velocity relation is found and is inserted in eqn(3.7)to find complex conductivity in eqn(3.8). This eqn is substituted in Maxwell solution in eqn (3.9). By assuming the wave number K to be equal to that of free space a useful expression for friction energy eqn (3.15) is found by using Plank quantum hypothesis, for non-polarized and polarized medium. It is very striking to note that this frictional energy expression is similar to that of eqn (4.1).Quantum Equation for frictional medium, which is reduced to ordinary schodingereqn is found in section 4 eqn (4.8).By treating particles as vibrating strings or moving in a circular crbit, a useful expression of friction energy is found. The friction energy is shown to be quantized.

CONCLUSION

Maxwell Equations for damping or non-damping electromagnetic wave, in the presence of friction can be used to derive new Schrodinger Equation; this equation reduces to ordinary Schrodinger Equation and shows quantized friction energy.

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